THE DYNAMIC COSMOLOGICAL TERM Λ: SOME ASPECTS OF PHENOMENOLOGICAL MODELS

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Abstract. Choosing a phenomenological model of Λ, viz. $\Lambda \sim \dot{H}$, it has been shown that this model of Λ is equivalent to other three types of Λ, $\Lambda \sim (\dot{a}/a)^2$, $\Lambda \sim \ddot{a}/a$ and $\Lambda \sim \rho$. Through an indirect approach, it has also been possible to put a limit on the deceleration parameter q. It has been shown that if q becomes less than -1, then this model can predict about the presence of phantom energy.

KEY WORDS: general relativity; cosmological parameters; phenomenological models; phantom energy.
PACS number(s): 98.80.Jk, 98.80.Cq.

1. Introduction

The experiments on supernovae type Ia have been provided conclusive evidences that the present expanding Universe is in accelerating phase (Perlmutter et al. 1998; Riess et al. 1998). Obviously, this suggests that a repulsive gravitation, in the form of some kind of exotic energy, is acting behind this unexpected phenomenon. The so-called cosmological constant Λ was thought to be a possible candidate of this exotic or dark energy. However, Λ was assumed as a dynamic term depending on the cosmic time so that it can explain the rate of acceleration.

As a consequence of the search for current status of this acceleration some phenomenological models of Λ, viz. $\Lambda \sim (\dot{a}/a)^2$, $\Lambda \sim \ddot{a}/a$ and $\Lambda \sim \rho$, recently we (Ray and Mukhopadhyay 2004) have shown the equivalence of these models. We also have established a relationship between the parameters α, β and γ of the respective models. It was mentioned that $\Lambda \sim \ddot{a}/a$ model can be viewed as a combination of $\Lambda \sim (\dot{a}/a)^2$ and $\Lambda \sim \dot{H}$ models (since, $\ddot{a}/a = (\dot{a}/a)^2 + \dot{H}$). Therefore, for $\dot{H} = 0$ the models $\Lambda \sim H^2$ and $\Lambda \sim \ddot{a}/a$ become identical, where $a(t)$ is the scale factor of the

Universe and $H(=\dot{a}/a)$ is the Hubble parameter. Since, $\Lambda \sim \ddot{a}/a$ model depends on H^2 and $\dot{H}$, and $\Lambda \sim H^2$ model has already been studied by us (Ray and Mukhopadhyay 2004), the point of interest is now $\Lambda \sim \dot{H}$ model. Although a number of phenomenological models have been listed by Overduin and Cooperstock (1998) (also see the references in Ray and Mukhopadhyay 2004) but $\Lambda \sim \dot{H}$ model is not included there.

In the present paper, Einstein's field equations have been solved for $a(t)$, $\rho(t)$ and $\Lambda(t)$ with the *ansatz* $\Lambda = \mu\dot{H}$, where μ is a parameter of $\Lambda \sim \dot{H}$ model. It has been shown that $\Lambda \sim \dot{H}$ model is equivalent to $\Lambda \sim (\dot{a}/a)^2$, $\Lambda \sim \ddot{a}/a$ and $\Lambda \sim \rho$ models when the solutions for a $a(t)$, $\rho(t)$ and $\Lambda(t)$ are expressed in terms of Ω_m and Ω_Λ, respectively the matter- and vacuum-energy densities of the Universe. It has also been possible to connect μ with α, β and γ. Section 5 deals with the physical implications and limit on the deceleration parameter q, for the present model while equivalence of $\Lambda \sim \dot{H}$ model with $\Lambda \sim (\dot{a}/a)^2$, $\Lambda \sim \ddot{a}/a$ and $\Lambda \sim \rho$ models as well as interrelationship of μ with α, β and γ are presented in Section 4. Sections 2 and 3 deals, respectively, with Einstein's field equations and their solutions. Finally, all the results and their significance are discussed in Section 6.

2. Einstein Field Equations

The Einstein field equations are given by

$$R^{ij} - \frac{1}{2}Rg^{ij} = -8\pi G\left[T^{ij} - \frac{\Lambda}{8\pi G}g^{ij}\right] \tag{1}$$

where $\Lambda = \Lambda(t)$ is the time dependent cosmological constant and c, the velocity of light in vacuum, is assumed to be unity in relativistic units. If we choose the spherically symmetric Friedmann-Lemaître-Robertson-Walker (FLRW) metric

$$ds^2 = -dt^2 + a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \tag{2}$$

where the curvature constant k can assume the values $-1, 0, +1$ respectively for open, flat and close Universe models, then for a flat Universe, equation (1) yields respectively the Friedmann equation and the Raychaudhuri equation which are given by

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G\rho + \Lambda, \tag{3}$$

$$3\frac{\ddot{a}}{a} = -4\pi G(\rho + 3p) + \Lambda. \tag{4}$$

Let us now consider the barotropic equation of state in the form

$$p = w\rho \tag{5}$$

where w, the equation of state parameter, for the dust, radiation, vacuum fluid and stiff fluid can take the constant values 0, 1/3, -1 and $+1$ respectively.

Let us assume that G does not vary with space and time. Then for the ansatz

$$\Lambda = \mu \dot{H} \tag{6}$$

where μ is a free parameter, we get from the equations (3) and (4) the following two modified equations

$$4\pi G\rho = \frac{1}{2}(3H^2 - \mu\dot{H}). \tag{7}$$

$$3(H^2 + \dot{H}) = -4\pi G\rho(1 + 3\omega) + \mu\dot{H} \tag{8}$$

Equations (7) and (8), on simplification, yield the differential equation

$$(2 - \mu - \mu\omega)\dot{H} = -3(1 + w)H^2. \tag{9}$$

3. The Models

Now, equation (9) on integration gives

$$H = \frac{2 - \mu - \mu\omega}{3(1 + w)t}. \tag{10}$$

Putting $H = \dot{a}/a$ in equation (10) and integrating it further we get our general solution set as

$$a(t) = Ct^{\frac{2-\mu-\mu\omega}{3(1+w)}}, \tag{11}$$

$$\rho(t) = \frac{2 - \mu - \mu\omega}{12\pi G(1 + w)^2}t^{-2}, \tag{12}$$

$$\Lambda(t) = \frac{-\mu(2 - \mu - \mu\omega)}{3(1 + w)}t^{-2} \tag{13}$$

where C is an integration constant.

(i) *Dust case* ($\omega = 0$)

For dust case, equations (11), (12), (13) and (10), respectively takes the form

$$a(t) = Ct^{(2-\mu)/3}, \tag{14}$$

$$\rho(t) = \frac{2-\mu}{12\pi G}t^{-2}, \tag{15}$$

$$\Lambda(t) = \frac{-\mu(2-\mu)}{3}t^{-2}, \tag{16}$$

$$t = \frac{2-\mu}{3H}. \tag{17}$$

Equation (15) suggests that for physically valid ρ (i.e. $\rho > 0$), we should get $\mu < 2$. So, μ can be negative as well. Again, from equation (16) we find that for a repulsive Λ, the constraint on μ is that it must be negative. Thus, equations (15), (16) and (17) all point towards a negative μ.

(ii) *Radiation case* ($\omega = 1/3$)

For radiation case, equations (11), (12), (13) and (10), respectively reduce to

$$a(t) = Ct^{(3-2\mu)/6}, \tag{18}$$

$$\rho(t) = \frac{3-2\mu}{32\pi G}t^{-2}, \tag{19}$$

$$\Lambda(t) = \frac{-\mu(3-2\mu)}{6}t^{-2}, \tag{20}$$

$$t = \frac{3-2\mu}{6H}. \tag{21}$$

Equation (19) suggests that for physically valid ρ, $\mu < 3/2$ whereas equation (20) demands a negative μ for repulsive Λ. Thus, in this case also a negative μ is necessary for equations (19) - (21).

4. Equivalent Relationship Between Λ-Dependent Models

Using the value of t from equation (17) in equation (15) we get

$$\Omega_m = \frac{2}{2-\mu}, \tag{22}$$

where $\Omega_m (= 8\pi G\rho/3H^2)$ is the matter-energy density of the Universe.

Again, using equation (17) we get from equation (16)

$$\Omega_\Lambda = \frac{-\mu}{2-\mu}, \tag{23}$$

where $\Omega_\Lambda (= \Lambda/3H^2)$ is the vacuum-energy density of the Universe.

Adding (22) and (23) we get,

$$\Omega_m + \Omega_\Lambda = 1 \tag{24}$$

which is another form of equation (3) for flat Universe. Also, using the value of μ from equation (22), we get from (17),

$$t = \frac{2}{3\Omega_m H}. \tag{25}$$

Thus, if t_0 and H_0 be the values of t and H at the present epoch, then

$$t_0 = \frac{2}{3\Omega_{m0} H_0}. \tag{26}$$

Equation (26) is the same expression for t_0 as obtained by us (Ray and Mukhopadhyay 2004) for $\Lambda \sim (\dot{a}/a)^2$, $\Lambda \sim \ddot{a}/a$ and $\Lambda \sim \rho$. Again, using equation (23) in equations (14)-(16), it can easily be shown that the expressions for $a(t)$, $\rho(t)$ $\Lambda(t)$ become identical with the expressions for those quantities as obtained by us (Ray and Mukhopadhyay 2004). This means that $\Lambda \sim \dot{H}$ model is equivalent to $\Lambda \sim (\dot{a}/a)^2$, $\Lambda \sim \ddot{a}/a$ and $\Lambda \sim \rho$. Again, from equation (22) we get,

$$\mu = -\frac{2\Omega_\Lambda}{\Omega_m}. \tag{27}$$

But, we (Ray and Mukhopadhyay 2004) already have shown that

$$\gamma = \frac{\Omega_\Lambda}{\Omega_m}. \tag{28}$$

Thus, equations (27) and (28) yield

$$\mu = -2\gamma. \tag{29}$$

Also, we (Ray and Mukhopadhyay 2004) have shown that for pressureless dust α, β, γ respectively the three parameters of $\Lambda \sim (\dot{a}/a)^2$, $\Lambda \sim \ddot{a}/a$ and $\Lambda \sim \rho$ models can be interconnected by the relation

$$\alpha = \frac{\beta}{3(\beta-2)} = \frac{\gamma}{\gamma+1}. \tag{30}$$

Thus, combining equation (29) with equation (30) we have,

$$\alpha = \frac{\beta}{3(\beta-2)} = \frac{\gamma}{\gamma+1} = \frac{\mu}{\mu-2}. \tag{31}$$

Similarly, it can be shown that for radiation-filled Universe, μ is related to Ω_m and Ω_Λ respectively by the relations,

$$\Omega_m = \frac{3}{3-2\mu}, \tag{32}$$

$$\Omega_\Lambda = \frac{-2\mu}{3-2\mu}. \tag{33}$$

Adding (32) and (33) we get again,

$$\Omega_m + \Omega_\Lambda = 1. \tag{34}$$

In this case, μ is related to γ of our previous investigation (Ray and Mukhopadhyay 2004) by

$$\mu = -\frac{3}{2}\gamma. \tag{35}$$

Using the value of μ from equation (32) in equation (21) we have,

$$t = \frac{1}{2\Omega_m H} \tag{36}$$

which for the present Universe yields

$$t_0 = \frac{1}{2\Omega_{m0} H_0}. \tag{37}$$

Again, equation (37) is the same expression for t_0 as obtained by us (Ray and Mukhopadhyay 2004) for $\Lambda \sim (\dot{a}/a)^2$, $\Lambda \sim \ddot{a}/a$ and $\Lambda \sim \rho$ models in radiation case. Also, combining equation (35) with equation (39) of our previous investigation (Ray and Mukhopadhyay 2004) we find that in the radiation case, μ is related to α, β, γ by

$$\alpha = \frac{\beta}{2\beta-3} = \frac{\gamma}{\gamma+1} = \frac{2\mu}{2\mu-3}. \tag{38}$$

5. Physical Features of the Models

The deceleration parameter q is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -(1 + \frac{\dot{H}}{H^2}). \tag{39}$$

Thus, using equation (10) we have

$$q = -\left[1 - \frac{3(1+\omega)}{2 - \mu - \mu\omega}\right]. \tag{40}$$

For an accelerating Universe, $q < 0$ and hence, $\mu < -(1+3\omega)/(1+\omega)$. Equation (40) tells us that for a dust-filled accelerating Universe, μ should be less than -1. We have already shown that for physically valid ρ, Λ and t, μ must be negative. Thus, for $\mu < -1$ we get an accelerating Universe with repulsive Λ through our $\Lambda \sim \dot{H}$ model.

Again, equation (39) can be written as

$$\dot{H} = -H^2(q+1) \tag{41}$$

so that one can find an expression for the variable cosmological constant as

$$\Lambda = -\mu H^2(q+1) \tag{42}$$

as $\Lambda = \mu\dot{H}$. If $\mu < 0$, then equation (42) tells us that Λ remains a repulsive force (i.e. $\Lambda > 0$ so long as $q > -1$. This means that if the acceleration of the Universe exceeds a certain limit then Λ will become an attractive force. In the present accelerating Universe, q lies near -0.5 (Overduin and Cooperstock 1998; Efstathiou 1998; Sahni 1999) and hence $(q+1) > 0$. Since, for our model both $\dot{H}$ and H^2 are proportional to t^{-2} then it is clear from equation (39) that q is a constant quantity. But through our model it has been possible to exhibit indirectly via equation (42) that in future Λ may become an attractive force provided q becomes less -1.

Discussions

It has been shown, in the present investigation, that $\Lambda = \mu\dot{H}$ model is equivalent to other three types of phenomenological models, viz. $\Lambda = \alpha(\dot{a}/a)^2$, $\Lambda = \beta\ddot{a}/a$ and $\Lambda = \gamma\rho$. Also, a relationship is established between the parameters μ and γ in both dust and radiation cases. It was observed in the previous investigation (Ray and Mukhopadhyay 2004) that α and γ are related to Ω_m

and Ω_Λ through the same relation in both dust and radiation cases while β is related to Ω_m and Ω_Λ through different relations as expression for β in terms of these cosmic energy densities contains ω. Since, $\ddot{a}/a = H^2 + \dot{H}$, then it is clear that dependency of the parameter ω on β is due to $\Lambda \sim \dot{H}$ part because relation of μ with Ω_m and Ω_Λ also contains ω and hence μ behaves differently with cosmic matter and vacuum energy density parameters in dust and radiation cases.

It has been possible through our model to put a limit on q, the deceleration parameter of the Universe. If one is content with the idea of a repulsive Λ only, then we find that q cannot exceed a certain limit. But, if one is bold enough to think of an attractive Λ in future, then q can go on decreasing indefinitely, i.e. the rate of acceleration can increase indefinitely. But in presence of an attractive Λ, indefinite increase in acceleration is possible only if G goes on decreasing with time. Now, an indefinite increase in acceleration may lead to 'Big Rip' (Caldwell et al. 2003) or 'Partial Rip' (Štefančić, 2004a) of the Universe for a super-negative (< -1) equation of state parameter known as phantom energy. Therefore, although we have worked out the present model with the assumption that G is constant, yet through an indirect approach it has been possible to predict about the presence of phantom energy. Moreover, recently Štefančić (2004b) has shown that cosmological model without phantom energy may exhibit the same effect as that of phantom energy. This is very significant because in the present work also without any *a priori* assumption regarding phantom energy, it is shown that in future the Universe may go on expanding with an ever increasing acceleration similar to that of phantom energy models if the value of the deceleration parameter drops below -1.

Finally, it should be mentioned that any linear combination of $\Lambda \sim (\dot{a}/a)^2$, $\Lambda \sim \ddot{a}/a$ and $\Lambda \sim \rho$ is also equivalent.

Acknowledgement

One of the authors (SR) would like to express his gratitude to the authorities of IUCAA, Pune for providing him the Associateship Programme under which a part of this work was carried out.

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